Hybrid Models of Airline Competition: Capacity Allocation Games with Embedded Airfare Regression

Xuemei LI[a,b], Vikrant Vaze[b]

[a] School of Economics and Management, Beijing Jiaotong University , 3 Shangyuancun, Beijing 100044, China

[b] Multi Decision-Maker Systems Group, Thayer School of Engineering, Dartmouth College, Hanover, NH 03755, United States

Abstract: Aiming at benefits, airfare is made by airlines considering the situation of its company, including micro-environment and macro-environment, such as company, market, other airlines' competitive strategy, competitive landscape, government policy and regulations, and other consideration. Usually, Airfare is a Nash Equilibrium solution to game among airlines as players who want the best outcome for themselves and take the actions of others into account. Because of market developing, to obtain great market share and optimize the benefit the players pay more attention to the strategy corresponding to other players, main factors affecting the outcome and benefit, and how and how much to affect the airfare. Thus, after discussing the factors related to airfare determine, this paper develops hybrid models of airline competition in capacity allocation games with embedded airfare regression under the game-theoretic and econometric model.

The hybrid model of airline competition includes multi-stage games, where each stage determines corresponding fare considering main influencing factors of airline in the specific market. The number of stages is determined by how many main airlines in the same market, and the sequence of stages depends on the competitiveness of airlines in the same market. The approximate results of accurate scenario analysis by developed model will contribute to airline strategic decision-support, the actual capacity allocation decision (frequency and seats) and airfare. Moreover, it will reveal the relationship respectively between airfare and cost (fuel price and labor expenses), economic influence, sum of passenger, distance, time zone, and even some interaction influence.

Keywords:

Hybrid Approach

Game-Theoretic and Econometric Model

Airfare Regression

Capacity Competition

1 Introduction

From the view of market, market share is competed among airlines who already have a market share and potentially about to enter the market of the company. Their sharing market is a typical dynamic process. The competition games are stable when Nash Equilibrium (NE) reaches which mean every airline (the play) should take other airline's (player's) strategy into account. So, the decision of market share and airfare is a game-theoretic question.

From the view of airline, airfare is affected by many factors, such as external factors that are macroeconomic, competitors, seasons, time zone, and so on; operating factors which are operating cost of labor, fuel, and so on, distance, passengers, and so on; and some consideration.

Therefore, airlines' capacity and fare decisions under competition over a network are modeled as a complicate game. Multi-stage competition game model for the market are put forward and every stage is an airfare regression model for a specific airline operation approximate.

2 Literature Review

2.1 Economic Model for air fare predict

From the view of economic, air fare decision is related to some factors…… (temporary absence)

2.2 Game Model for Airline Competition

2.2.1 Airline Planning Literature

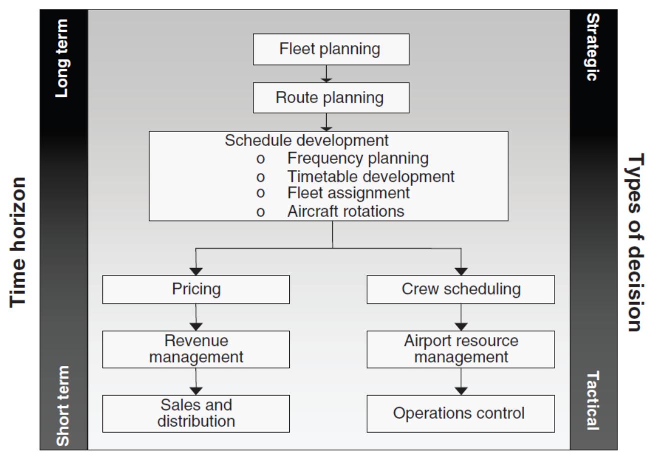

Fig 1: Overview of the Airline Planning Process (Belobaba, 2015b)

Many parts of the airline planning process (Fig. 1) have been extensively studied in literature. Airline planning process involves deciding the set of flights to be operated, the set of aircraft and crew to operate those flights, and the set of fares on those flights. These decisions are typically made sequentially. Long-term decisions about Fleet Planning and Route Planning are made before medium-term Schedule Development decisions. Fleet Planning is the long-term strategic process of deciding an airline's fleet of aircraft, while Route

Planning is the subsequent process of deciding the set of routes (or airport pairs) to offer flights on. Though fleet and route planning decisions are made using various analytical methods based on a variety of technical, economic, financial and other considerations (Belobaba, 2015b), advanced operations research (OR) models for these two steps are rarely used in practice and rarely studied in OR literature, despite their obvious connection with ACA decisions. This paper does not focus on fleet and route planning decisions. At the other end of the spectrum, short-term tactical decisions including, Sales and Distribution, Airport Resource Management, and Operations Control are not related to ACA decisions, and hence this project does not focus on them. This paper focuses on medium-term decisions of Schedule Development (SD), and Pricing and Revenue Management (PRM).

Within Schedule Development, the four sub-steps – Frequency Planning, Timetable Development, Fleet Assignment and Aircraft Routing (rotations) – are typically performed sequentially. They are followed by the Crew Scheduling step, which is itself divided into two sub-steps called Crew Pairing and Crew Rostering. Existing literature includes sophisticated optimization-based decision-support tools and models for Fleet Assignment, Aircraft Routing and Crew Scheduling, but has a very limited coverage of Frequency Planning, and Timetable Development problems.

Aircraft Routing and Crew Scheduling problems have been extensively studied and mixed-integer optimization models have been developed for minimizing operating costs to find feasible aircraft and crew itineraries (Arabeyre, Fearnley, Steiger, and Teather, 1969; Barnhart et al., 1998; Cordeau, Stojkovic, Soumis, and Desrosiers, 2001; Klabjan, Johnson, and Nemhauser, 2001; Cohn and Barnhart, 2003; Saddoune, Desaulniers, Elhallaoui, and Soumis, 2012). Aircraft Routing and Crew Scheduling decisions, however, do not affect ACA decisions; do not affect passenger demand and revenues; and do not interact with competitors’ decisions. Therefore, they are not directly related with this project.

Frequency Planning, Timetable Development and Fleet Assignment problems, in contrast, are directly related to the ACA decisions. Many prior studies address the fleet assignment problem, sometimes also integrated with Aircraft Routing and Crew Scheduling problems. Hane et al. (1995) provided a fleet assignment optimization model assuming fixed passenger demand for each flight leg. Their model was subsequently extended by Barnhart, Kniker, and Lohatepanont (2002) to account for passenger connections, and also for passenger spill and recapture phenomena. This model was further enhanced to capture passenger itinerary choice in recent studies (Atasoy, Salani, and Bierlaire, 2014; and Wang, Klabjan, and Shebalov, 2014). While no study in prior literature has tackled the stand-alone Timetable Development optimization problem, some prior studies have incorporated incremental timetable decisions into the fleet assignment problems, either through minor adjustments in flight departure times using time-windows (e.g., Desaulniers et al., 1997; Rexing et al., 2000; Sherali, Bae, and Haouari, 2013) or through binary decisions about whether or not to operate some optional flight legs (e.g., Lohatepanont and Barnhart, 2004; Sherali, Bae, and Haouari, 2010; Atasoy, Salani, and Bierlaire, 2014; and Wang, Klabjan, and Shebalov, 2014). However, none of these studies explicitly incorporate frequency planning decisions. Pita, Barnhart, and Antunes (2013) and Cadarso, Vaze, Barnhart, and Marin (2015) are the only two studies that incorporate frequency decisions into other schedule development problems. Both these studies jointly optimize frequency planning, approximate timetable development and fleet assignment, and thus directly deal with the ACA decisions.

Pricing and Revenue management (PRM) focus on optimizing revenue from the sale of available flight seating capacity. Over the last few decades, various heuristic and exact methods have been developed and applied for optimizing PRM decisions (e.g., Belobaba, 1989; McGill and Van Ryzin, 1999; Clough, Jacobs, and Gel, 2014). For excellent review of the methods, concepts and literature in this area, please refer to Talluri and van Ryzin (2004) or Belobaba (2015a). Despite the interdependence between PRM and SD decisions, OR literature on PRM optimization does not overlap with SD optimization literature (except, to a limited extent, in some studies like Jacobs, Smith and Johnson (2008)). In addition to algorithmic tractability concerns associated with solving the resulting giant integrated optimization problem, there are two main reasons for this separation. First, the timelines for the two tasks are very different with most of the SD decisions made several months or weeks ahead of the flight departure time and PRM decisions made a few weeks to a few minutes ahead of the time of the flight departure time. Second, within an airline’s organization, PRM departments operate almost independently of the SD departments. Therefore, the state-of-the-art scheduling models typically assume fixed fares per flight, per itinerary, or per passenger segment. These assumed fares reflect their best judgment of the true fare levels to be expected, which is necessarily an approximation. Similarly, state-of-the-art PRM models assume pre-specified flight frequency and seats-per-flight, which in turn are previously determined based on an approximate understanding of the fare values to be expected. Thus, ACA decisions and PRM decisions are interrelated, but follow different timelines and are made by different departments in an airline organization. Therefore, this project focuses on ACA and PRM decisions in a two-stage model.

2.2.2 Airline Competition Literature

None of the studies from the airline planning literature above address game-theoretic aspects of airline competition. There is a separate stream of studies focusing on airline competition. These studies apply the concept of Nash equilibrium (or its refinements, approximations or generalizations) to find a solution of the airline competition game. These studies can be classified into two different categories: *generic* and *airline-specific*. Generic category includes studies using general economic models (such as Cournot, Bertrand, and Stackelberg models) of oligopoly competition between firms based of quantity and prices (e.g., Brander and Zhang, 1993; Norman and Strandenes, 1994; Hendricks, Piccione, and Tan, 1999; Brueckner, 2002; Brueckner and Flores-Fillol, 2007; and Aguirregabiria and Ho, 2010). *Airline-specific* category includes models customized for airline competition context and hence is relevant for this paper.

In the airline-specific competition studies, market share is usually represented by a logit-type model (such as multinomial logit or nested logit) with the passenger utility being primarily a function of flight frequency and average fare. Hansen (1990) was the first to solve for an equilibrium of an airline frequency competition game using real-world data. He solved airline profit optimization problems repeatedly till near-convergence to a single-stage frequency equilibrium for a large case study with 52 U.S. airports and 28 airlines. In a series of studies especially focusing on the European aviation network (Adler, 2001; 2005; Adler and Berechman, 2001; Adler and Smilowitz, 2007; Adler, Pels, and Nash, 2010), Adler and colleagues developed – in some cases as a standalone single-stage game, and in other cases as the second stage following a route planning stage in a two-stage game – a simultaneous equilibrium approach for frequency, fare, and seat decisions. More recently, Vaze and Barnhart (2012a; 2012b; and 2015) solved a single-stage frequency competition game to obtain a Nash equilibrium. A few other studies have also developed and solved single-stage game models to a Nash equilibrium involving simultaneous frequency, fare and seat decisions (Brueckner, 2010), simultaneous frequency and fare decisions while holding seats-per-flight constant (Hong and Harker, 1992; Pels, Nijkamp, and Rietveld, 2000; and Hansen and Liu, 2015), simultaneous frequency and seat decisions while holding fares constant (Wei and Hansen, 2007), or fare decisions while holding frequency and seats constant (Aguirregabiria and Ho, 2012).

Many of these studies (e.g., Dobson and Lederer, 1993; Norman and Strandenes, 1994; Schipper, Rietveld, and Nijkamp, 2003; Brueckner and Flores-Fillol, 2007; Hansen and Liu, 2015; Vaze and Barnhart, 2015) have noted the need for developing and solving two-stage game models with the first stage consisting of frequency and/or seats-per-flight decisions, and the second stage consisting of fare decisions, in order to be consistent with the actual sequence in which decisions are made in the airlines. However, the analytical and computational treatment of two-stage capacity-fare games is very limited in the existing literature, and focuses exclusively on single-market, two-airline games. Dobson and Lederer (1993) acknowledge the tractability issues and make several simplifying assumptions (including assuming a single aircraft size across the network) and use heuristic methods for solving the single-market, two-airline game. Schipper, Rietveld, and Nijkamp (2003) analyze the change in passenger welfare due to deregulation by analyzing the shift from monopoly to duopoly equilibria for a single-market, two-airline game by focusing only on symmetric equilibria. Brueckner and Flores-Fillol (2007) provide a brief discussion of the comparative statics of the single-market, two-airline game through two propositions that compare the results with the corresponding single-stage game. Hansen and Liu (2015) provide a small numerical example of single-market, two-airline game. Norman and Strandenes (1994) and Vaze and Barnhart (2015) only focus on the one-stage game. In summary, though behaviorally consistent with the actual airline process, all existing studies on two-stage capacity-fare games focus on single-market, two-airline games; do not provide any tractability results for handling larger, more realistic networks; do not model passenger connections; do not address decisions about seats-per-flight; and do not provide any empirical validation of their results. This paper addresses all of these challenges.

3 Multi-Stage Hybrid Model of Airline Competition

Here airlines' capacity and fare decisions under competition over a network are modeled as a multi-stage game. Consistent with the actual airline planning process, the capacity allocation decisions (i.e., flight frequency and seats on each non-stop flight segment) are made every stage of the game deals with the approximate decisions about average fare in each market. This section describes the framework using the terms market, airline and time. A market is defined as an ordered pair of airports between which passengers wish to travel, an airline is defined as a carrier operating in the market, and time is defined as a compute cycle. Therefore, to estimate the airfare airline decision, here creates a hybrid model of airline competition with multi-stage games involving capacity and pricing competition, where Two-Stage Least Square Analysis (2SLS) is used for analysis of structural equations, and embedded airfare regression is used for specific airline. The general model express as follow,

$$\begin{cases} \widehat{y_1} = f_1(Ex) \\ \widehat{y_2} = f_2(Ex, y_1) \\ \widehat{y_3} = f_3(Ex, y_1, y_2) \\ \dots\dots \end{cases} \tag{1}$$

where $\widehat{y_i}$ $(i = 1,2, \dots.)$ are endogenous variables representing the approximate airfare (yield: revenue per passenger mile RPM) of ranking i carrier (player) in the specific market which is determined within the system of equations; y_i $(i = 1,2, \dots.)$ are exogenous variables representing the real airfare of ranking i carrier (player); Ex are exogenous variables representing affecting factors of airfare. Within the simultaneous equation every equation represents a sub-game among the whole competitive market. The number of equations, similarly, the number of sub-games is determined by how many main carriers there are in one market. From this model, we know that the ranking first carrier has priority of determining its airfare in the market, then the ranking second carrier turn, the ranking third carrier turn, etc. Each equation is expressed as a linear combination of fare including economy factors, carrier, cost, time, market share, and capacity, some transformation of factors, and possibly some other factors.

Some notations are described next. Let $ECON$ be economic factors, $CARRIER$ the airline, $COST$ be flight operating cost, $TIME$ be compute cycle, $MARKET\_SHARE$ be airline market share in a market, $CAPACITY$ be flight frequency and seats. The general form fare decision model of each carrier in specific market with a compute cycle is denoted by:

$$AIRFARE \sim ECON + CARRIER + COST + TIME + MARKET\_SHARE + CAPACITY \quad (2)$$

where $AIRFARE$ means $\widehat{y_i}$ as an objective variable in the airline-specific competition studies, $ECON + CARRIER + COST + TIME + MARKET\_SHARE + CAPACITY$ means a linear combination of the main factors as explanatory variables for approximate $AIRFARE$.

Moreover, the image of hybrid model on airline competition can be described as follow (Fig.2),

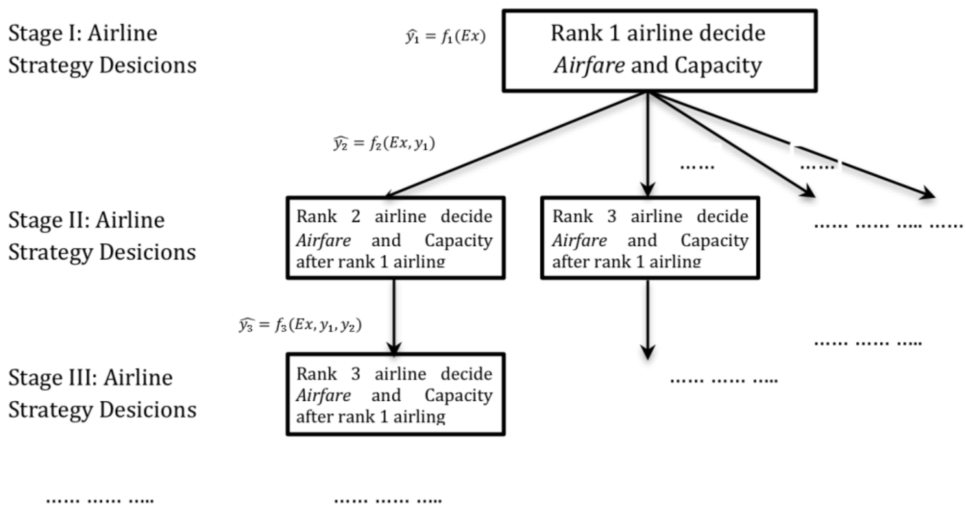

Fig. 2 Multi-Stage Modeling Image

4 Case Study for Non-stop Domestic Market

Analytical development for hybrid model of airline competition implements with simplified versions of real-world domestic networks and carriers, including networks consisting of 35 airports (OEP35), 5 main airlines (WN, DL, UA, AA, and US).

4.1 Data for Case Study

The Hybrid model involves many kinds of factors from deferent databases which are publicly available from BTS (Bureau of Transportation Statistics) and BEA (Bureau of Economic Analysis), including:

- **Airline Origin and Destination Survey (DB1B) database:** This database, which is used to determine air traffic patterns, air carrier market shares and passenger flows, is a 10% sample of airline tickets from reporting carriers collected by the Office of Airline Information of the Bureau of Transportation Statistics. From DB1B the aggregated quarterly data (from the first quarter of 1993 to the fourth quarter of 2014) are downloaded and filtered, such as airfare each market from origin to destination (OEP35 airport, the 35 major US airports) by unique carrier (five main airline, Southwestern, Delta, United, American, and US airway), passengers (sum of passengers per quarter each market), distance (the mileages each market).
- **Air Carrier Financial Schedule P-12(a) Database:** This database contains monthly reported fuel costs, and gallons of fuel consumed, by air carrier and category of fuel use, including scheduled and non-scheduled service for domestic and international traffic regions. After processing the downloaded data, the quarterly fuel price per gallons can be obtained.

- **Air Carrier Financial Schedule P-5.2 Database:** This database contains detailed quarterly aircraft operating expenses for large certificated U.S. air carriers. It includes information such as flying expenses for maintenance of flight equipment, equipment depreciation costs, and total operating expense. After filtering, the cost of labor for flight operating can be obtained.

- **Air Carrier Financial** T-100 Domestic Segment database: This database contains domestic and international airline market and segment data which frequently used by the aviation industry, the press, and the legislature to produce reports and analyses on air traffic patterns, carrier market shares, as well as passenger, freight, and mail cargo flow within the aviation mode. After processing the downloaded data, flight frequency and seats can be obtained.
- **National Economic Accounts Database:** From this database, GDP percent of change from preceding period data can be download. After processing the data, all GDP change data are obtained quarterly.

Using market, carrier, and time data, all data selected from different database are merged.

4.2 Airfare Regression Model for Aggregate Data

Five airlines AA, DL, UA, US, and WN are selected in this equation, and thirty-five top airports from OEP35 are listed here. After checking the official data from US Department Of Transportation, here selects data span across a time of 88 quarters from the first quarter of 1993 to the fourth quarter of 2014. Each observation is the combination of a quarter, airline, origin and destination. There are over 130,000 observations after filtering in the dataset. Lots of various models specifications are tried and found the following model work well due to the R-Square and p-value.

Given a general regression equation in the simultaneous equation:

$$
\begin{aligned}
logAIRFARE(t) \sim & logFUEL(t-1) + logLABOR + GDP\_CHANGE + QUARTER1to4 + QUARTER2to4 + QUARTER3to4 \\
& + AAtoWN + DLtoWN + UAtoWN + UStoWN + TIMEZONE\_absdiff\_1to0 + TIMEZONE\_absdiff\_2to0 \\
& + TIMEZONE\_adbsiff\_3to0 + DIST1 + DIST2 + DIST3 + PASSENGERS\_SUM + FREQUENCY \\
& + FREQ\_OTHERS + SEATS + logFUEL(t-1)\_AA + logFUEL(t-1)\_DL + logFUEL(t-1)\_UA \\
& + logFUEL(t-1)\_US + logLABOR\_AA + logLABOR\_DL + logLABOR\_UA + logLABOR\_US
\end{aligned}
\tag{2}
$$

where $AIRFARE(t)$ is the dependent variable that is air fare in cents per mile (also called yield). Because $AIRFARE(t)$ and the main explanatory numerical variables $FUEL(t-1)$ and $LABOR$ don't have stationary relationship, logarithmic transformation of those three variables is used to express their relationship well. Most importantly, since the log of airfare is a linear function of log of fuel prices and labor consumption, the model is easy to interpret. The model coefficient directly gives us the percentage change in fares when the fuel price changes by 1%, or the labor changes by 1%. Additionally, $QUARTER$, $CARRIER$ ($AtoWN, DLtoWN, UAtoWN\ and\ UStoWN$), and $TIMEZONE$ are dummy variables, and $DIST$ is piecewise linear function.

Explanatory variables in our model are:

- $FUEL(t-1)$ is fuel price in cents per gallon during the previous quarter. After trying many specifications, the lag of 1 quarter seemed to work the best.
- $LABOR$ contains the sum of all kinds of labor investment of the predicted carrier quarterly for airline operation and flight operating.
- GDP_CHANGE is percent change quarterly from preceding period which stands for the economic effect.
- $QUARTER1to4, QUARTER2to4\ and\ QUARTER3to4$ are quarter dummy variables, which give the quarter effects compared to the fourth quarter.
- $AtoWN, DLtoWN, UAtoWN\ and\ UStoWN$ are carrier dummy variables, which are the carrier compared to WN carrier.
- $TIMEZONE\_absdiff\_1to0, TIMEZONE\_absdiff\_2to0, TIMEZONE\_adbsiff\_3to0$ are time zone dummy variables, which are absolute time zone differences between origin and destination compared to zero time zone.
- $DIST1, DIST2, DIST3$ represent a piecewise linear function of distance with NONSTOP domesticmarket from origin endpoint to destination endpoint. The breakpoints in the piecewise linear relationships are at 500 miles and 1500 miles.
- $PASSENGERS\_SUM$ is the sum of different airline transporting passengers in the same quarter.

- $FREQUENCY$, same as scheduled departures which are takeoffs scheduled of the expected airline at an airport quarterly, as set forth in published, are the sum of flights per quarter.
- $FREQ\_OTHERS$ are the sum of takeoffs scheduled at an airport quarterly except the predicted airline.
- $SEATS$ are the sum of installed seats of the expected airline in an aircraft (including seats in lounges) exclusive of any seats not offered for sale to the public by the carrier within one quarter; provided that in no instance shall any seat sold be excluded from the count of available seats.
- $logFUEL(t-1)\_AA, logFUEL(t-1)\_DL, logFUEL(t-1)\_UA, logFUEL(t-1)\_US$ are interactions between carrier dummy variables and $logFUEL(t-1)$.
- $logLABOR\_AA, logLABOR\_DL, logLABOR\_UA, logLABOR\_US$ are interactions between carrier dummy variables and $logLABOR$.

The aggregate data, 36,421 observations, are taken into account, where the observations are filtered in terms of more than 90% market share carried by AA, DL, UA, US and WN. Using equation (2). The results are:

Coefficients: (1 not defined because of singularities)

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	2.032e+00	3.479e-02	58.394	< 2e-16	***
logFUEL(t-1)	2.337e-01	1.647e-02	14.186	< 2e-16	***
logLABOR	4.650e-01	3.985e-02	11.668	< 2e-16	***
PASSENGERS_SUM	-2.879e-05	5.275e-07	-54.581	< 2e-16	***
GDP_CHANGE	5.956e-03	5.849e-04	10.182	< 2e-16	***
QUARTER1to4	1.258e-02	4.139e-03	3.040	0.002364	**
QUARTER2to4	3.130e-02	4.053e-03	7.724	1.16e-14	***
QUARTER3to4	6.514e-03	4.052e-03	1.607	0.107972	
AAtoWN	1.727e+00	8.467e-02	20.399	< 2e-16	***
DLtoWN	1.674e+00	5.128e-02	32.647	< 2e-16	***
UAtoWN	8.399e-01	6.177e-02	13.598	< 2e-16	***
UStoWN	1.101e+00	1.044e-01	10.548	< 2e-16	***
TIMEZONE_absdiff_1to0	5.022e-02	3.569e-03	14.068	< 2e-16	***
TIMEZONE_absdiff_2to0	7.497e-02	7.328e-03	10.231	< 2e-16	***
TIMEZONE_absdiff_3to0	-1.146e-01	1.080e-02	-10.611	< 2e-16	***
DIST1	-2.901e-03	1.662e-05	-174.564	< 2e-16	***
DIST2	-8.944e-04	9.342e-06	-95.748	< 2e-16	***
DIST3	3.196e-04	1.383e-05	23.107	< 2e-16	***
FREQUENCY	4.205e-04	1.256e-05	33.474	< 2e-16	***
FREQ_OTHERS	2.255e-07	3.109e-06	0.073	0.942185	
SEATS	-1.450e-06	8.304e-08	-17.456	< 2e-16	***
logFUEL(t-1)_AA	-1.108e-01	1.707e-02	-6.492	8.61e-11	***
logFUEL(t-1)_DL	-6.798e-02	1.678e-02	-4.051	5.11e-05	***
logFUEL(t-1)_UA	-3.108e-03	1.698e-02	-0.183	0.854777	
logFUEL(t-1)_US	-5.813e-02	1.887e-02	-3.081	0.002065	**
logLABOR_AA	-3.224e-01	4.904e-02	-6.573	5.01e-11	***
logLABOR_DL	-3.806e-01	4.173e-02	-9.121	< 2e-16	***
logLABOR_UA	-1.487e-01	4.359e-02	-3.412	0.000645	***
logLABOR_US	-1.277e-01	4.562e-02	-2.800	0.005120	**

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2743 on 36392 degrees of freedom
Multiple R-squared: 0.8206, Adjusted R-squared: 0.8204

F-statistic: 5944 on 28 and 36392 DF, p-value: < 2.2e-16

- The model gives low p values for almost all independent variables, has a decent fit, and adjusted R square 0.8204 is high enough to verify the linear relationship between logAIRFARE and all explanatory variables. Also, fare has linear relationship with 1-quarter lag of fuel. The coefficient of logFUEL(t-1) is 0.2337 that means fare may increase 0.2337% when fuel price increases by 1%. But this is only for WN airline. When this result combined with the airline interaction variables, depending on airline, increase 1%

in fuel price results in, somewhere between 0.1229% increase in fare for AA, 0.1657% increase in fare for DL, 0.2306% increase in fare for UA and 10.6% increase in fare for US, and 0.2337% increase in fare for WN.

- The coefficient of logLABOR is 0.4650 that means fare may increase 0.465% when LABOR increases by 1%. But this is only for WN airline. When this result combined with the airline interaction variables, depending on airline, increase 1% in LABOR results in, somewhere between 0.0844% increase in fare for DL, 0.1426% increase in fare for AA, 0.3163% increase in fare for UA and 0.3373% increase in fare for US, and 0.465% increase in fare for WN.
- All else being equal the fares show an extremely slow increase compare to GDP changes. The coefficient of GDP_CHANGE is 0.005956 which means the fare will increase 0.005956% when GDP change goes up 1%. It's even lower than inflation increase.
- Compared to the fourth quarter, the fare of the second quarter is highest, followed by the first quarter, but fare for the third quarter is lower than the fourth quarter.
- The coefficient of AA, DL, UA and US are all positive compared to WN which implies the fare of WN carrier is the lowest, followed by UA, US, DL and AA. However, the fare of WN is also most sensitive to fuel price among all five carriers, then followed by UA, US, DL and AA via coefficients of interaction variables, which represents the same order.
- All else being equal the yield generally drops with the distances, but the rate of decrease itself decreases with distance. This is consistent with our understanding of airfare yields.
- All else being equal, as the passenger flow increasing, the fares first rise and then fall slightly. Also, the variables are statistically significant at the 1% level.
- Time zone differences are significant at 1%. It implies that the fare is highest for time zone differences of 2 and 1 when compared with those of 0, 3 and above.
- As the coefficient of FREQUENCY is $4.405*10^{-4}$, fare will increase by 3.86 cents per mile when adding 1 flight per day.
- As the coefficient of FREQ_OTHERS is $2.255*10^{-7}$, fare will increase by 0.002 cents per mile when adding 1 flight per day. That means other airline's flight schedule is not sensitive to the airfare.
- As the coefficient of SEATS is $-1.450*10^{-6}$, fare will decrease by 0.13 cents per mile when adding 10 seats per day.

4.3 Hybrid Model of Airline Competitions for Multi-Stage Game

Depending on the number of main carrier in the same market, the multi-stage games have different stages. After summarizing the domestic market via real-world data, three sorts of scenarios will be taken into consideration. There are lots of different airlines as carriers at different airports, but only a few of airlines are the dominant carries, while 60% of endpoints has one main carrier, 32% of endpoints has two main carriers, and 7% of endpoints has three carriers via data from DOT. Usually, the ranking first airline as one player in the markets has monopoly situation, and the airfare is decided by it, then the second, the third, etc. Thus, three sorts of scenarios are discussed and focus on the influence of frequency and seats on airfare.

(1) One Main Carrier Market

As for the one main carrier market, the airline has monopoly for pricing in that market. So, only one airfare regression equation is used to determine the airfare of the market. There are 14,847 observations in the dataset, the results are:

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	2.414e+00	4.575e-02	52.778	< 2e-16	***
logFUEL(t-1)	2.502e-01	2.048e-02	12.213	< 2e-16	***
logLABOR	3.409e-01	4.988e-02	6.834	8.58e-12	***
PASSENGERS_SUM	-3.280e-05	9.987e-07	-32.843	< 2e-16	***
GDP_CHANGE	4.546e-03	8.475e-04	5.365	8.23e-08	***
QUARTER1to4	2.813e-02	5.858e-03	4.802	1.59e-06	***
QUARTER2to4	4.018e-02	5.745e-03	6.993	2.81e-12	***
QUARTER3to4	1.174e-02	5.754e-03	2.041	0.041289	*
AAtoWN	2.038e+00	1.876e-01	10.860	< 2e-16	***
DLtoWN	1.757e+00	6.832e-02	25.714	< 2e-16	***
UAtoWN	8.377e-01	9.988e-02	8.388	< 2e-16	***
UStoWN	1.130e+00	1.278e-01	8.840	< 2e-16	***
TIMEZONE_absdiff_1to0	3.848e-02	5.452e-03	7.059	1.75e-12	***
TIMEZONE_absdiff_2to0	1.496e-01	1.323e-02	11.302	< 2e-16	***
TIMEZONE_absdiff_3to0	-8.871e-02	1.338e-02	-6.629	3.49e-11	***
DIST1	-3.029e-03	2.121e-05	-142.817	< 2e-16	***
DIST2	-9.326e-04	1.575e-05	-59.203	< 2e-16	***
DIST3	-1.501e-04	3.839e-05	-3.910	9.27e-05	***
FREQUENCY	1.454e-04	1.849e-05	7.863	4.02e-15	***
FREQ_OTHERS	-2.825e-05	4.643e-06	-6.085	1.19e-09	***
SEATS	-4.065e-07	1.087e-07	-3.740	0.000184	***
logFUEL(t-1)_AA	-1.732e-01	2.274e-02	-7.616	2.77e-14	***
logFUEL(t-1)_DL	-1.098e-01	2.093e-02	-5.244	1.59e-07	***
logFUEL(t-1)_UA	-5.935e-02	2.188e-02	-2.713	0.006684	**
logFUEL(t-1)_US	-1.072e-01	2.333e-02	-4.597	4.33e-06	***
logLABOR_AA	-3.335e-01	8.101e-02	-4.117	3.86e-05	***
logLABOR_DL	-2.960e-01	5.243e-02	-5.646	1.67e-08	***
logLABOR_UA	-4.639e-02	5.797e-02	-0.800	0.423622	
logLABOR_US	-8.121e-03	5.672e-02	-0.143	0.886155	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2489 on 14818 degrees of freedom
Multiple R-squared: 0.8747, Adjusted R-squared: 0.8745

F-statistic: 3696 on 28 and 14818 DF, p-value: < 2.2e-16

According to the results,

- As the coefficient of FREQUENCY is $1.454*10^{-4}$, fare will increase by 1.832 cents per mile when adding 1 flight per day.
- As the coefficient of FREQ_OTHERS is $-2.825*10^{-5}$, fare will decrease by 0.25 cents per mile when adding 1 flight per day.
- As the coefficient of SEATS is $-4.065*10^{-7}$, fare will decrease by 0.037 cents per mile when adding 10 seats per day.

(2) Two Main Carrier Market

There are 7,880 observations in the dataset, the results are:

Stage 1:

```
Coefficients:
                              Estimate Std. Error t value Pr(>|t|)
(Intercept)                  1.417e+00  7.254e-02  19.528   < 2e-16 ***
logFUEL(t-1)_R1              1.221e-01  3.830e-02   3.189 0.001435 **
logLABOR_R1                  8.744e-01  9.211e-02   9.494   < 2e-16 ***
PASSENGERS_SUM              -3.380e-05  9.980e-07 -33.870   < 2e-16 ***
GDP_CHANGE                   4.943e-03  1.151e-03   4.295 1.77e-05 ***
QUARTER1to4                 -1.471e-04  8.201e-03  -0.018 0.985689
QUARTER2to4                  3.214e-02  7.962e-03   4.036 5.48e-05 ***
QUARTER3to4                 -8.497e-04  7.950e-03  -0.107 0.914893
TIMEZONE_absdiff_1to0        3.929e-02  6.890e-03   5.703 1.22e-08 ***
TIMEZONE_absdiff_2to0       -2.516e-02  1.208e-02  -2.083 0.037313 *
TIMEZONE_absdiff_3to0       -1.264e-01  2.291e-02  -5.518 3.54e-08 ***
DIST1                       -2.835e-03  3.637e-05 -77.938   < 2e-16 ***
DIST2                       -7.567e-04  1.854e-05 -40.815   < 2e-16 ***
DIST3                        4.950e-05  3.364e-05   1.471 0.141242
FREQUENCY_R1                 6.231e-04  2.437e-05  25.573   < 2e-16 ***
SEATS_R1                    -2.100e-06  1.628e-07 -12.903   < 2e-16 ***
AAtoWN_R1                    1.664e+00  1.583e-01  10.511   < 2e-16 ***
DLtoWN_R1                    2.220e+00  9.915e-02  22.386   < 2e-16 ***
UAtoWN_R1                    1.352e+00  1.175e-01  11.509   < 2e-16 ***
UStoWN_R1                    9.611e-01  2.657e-01   3.617 0.000300 ***
logFUEL(t-1)_AA_R1           9.483e-04  3.922e-02   0.024 0.980709
logFUEL(t-1)_DL_R1           4.635e-02  3.878e-02   1.195 0.232033
logFUEL(t-1)_UA_R1           9.448e-02  3.907e-02   2.418 0.015630 *
logFUEL(t-1)_US_R1           1.723e-01  4.574e-02   3.768 0.000166 ***
logLABOR_AA_R1              -4.892e-01  1.068e-01  -4.580 4.73e-06 ***
logLABOR_DL_R1              -7.970e-01  9.465e-02  -8.420   < 2e-16 ***
logLABOR_UA_R1              -5.354e-01  9.749e-02  -5.492 4.11e-08 ***
logLABOR_US_R1              -5.482e-01  1.081e-01  -5.073 4.00e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2503 on 7852 degrees of freedom
Multiple R-squared:  0.7896,	Adjusted R-squared:  0.7889
F-statistic:  1091 on 27 and 7852 DF,  p-value: < 2.2e-16
```

For rank first airline:

- As the coefficient of FREQUENCY is $6.231*10^{-4}$, fare will increase by 5.77 cents per mile when adding 1 flight per day.
- As the coefficient of SEATS is $-2.1*10^{-6}$, fare will decrease by 0.19 cents per mile when adding 10 seats per day.

Stage 2:

```
Coefficients:
                              Estimate Std. Error t value Pr(>|t|)
(Intercept)                  4.314e-01  3.300e-02  13.075  < 2e-16 ***
logAIRFAIR(t)_R1             8.356e-01  5.016e-03 166.580  < 2e-16 ***
logFUEL(t-1)_R2             -1.198e-02  1.517e-02  -0.790  0.42964
logLABOR_R2                  9.279e-02  3.623e-02   2.561  0.01045 *
PASSENGERS_SUM              -3.110e-06  6.402e-07  -4.859 1.21e-06 ***
GDP_CHANGE                   1.135e-03  5.317e-04   2.134  0.03289 *
QUARTER1to4                 -6.501e-03  3.786e-03  -1.717  0.08605 .
QUARTER2to4                 -8.935e-03  3.685e-03  -2.425  0.01534 *
QUARTER3to4                 -2.874e-03  3.679e-03  -0.781  0.43476
TIMEZONE_absdiff_1to0       -8.376e-03  3.411e-03  -2.456  0.01408 *
TIMEZONE_absdiff_2to0        6.603e-03  5.727e-03   1.153  0.24889
TIMEZONE_absdiff_3to0       -2.024e-02  1.057e-02  -1.915  0.05550 .
DIST1                       -4.853e-04  2.028e-05 -23.925  < 2e-16 ***
DIST2                       -1.377e-04  9.010e-06 -15.278  < 2e-16 ***
DIST3                       -2.099e-05  1.605e-05  -1.308  0.19098
FREQUENCY_R2                 2.212e-04  2.313e-05   9.563  < 2e-16 ***
FREQ_OTHERS_R2              -3.249e-05  3.871e-06  -8.395  < 2e-16 ***
SEATS_R2                    -1.023e-06  1.779e-07  -5.753 9.12e-09 ***
AAtoWN_R2                    1.133e-01  6.462e-02   1.753  0.07965 .
DLtoWN_R2                   -5.709e-02  5.399e-02  -1.057  0.29042
UAtoWN_R2                   -3.007e-01  5.383e-02  -5.586 2.40e-08 ***
UStoWN_R2                    1.252e-01  9.990e-02   1.253  0.21029
logFUEL(t-1)_AA_R2           1.521e-02  1.559e-02   0.975  0.32943
logFUEL(t-1)_DL_R2           8.260e-02  1.569e-02   5.264 1.45e-07 ***
logFUEL(t-1)_UA_R2           1.179e-01  1.565e-02   7.535 5.44e-14 ***
logFUEL(t-1)_US_R2           7.385e-02  1.757e-02   4.203 2.66e-05 ***
logLABOR_AA_R2              -3.208e-02  4.195e-02  -0.765  0.44437
```

logLABOR_DL_R2	-7.202e-02	3.928e-02	-1.833	0.06679	.
logLABOR_UA_R2	-1.138e-02	3.917e-02	-0.291	0.77132	
logLABOR_US_R2	-1.261e-01	4.201e-02	-3.002	0.00269	**

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1157 on 7850 degrees of freedom
Multiple R-squared: 0.9578, Adjusted R-squared: 0.9576

F-statistic: 6139 on 29 and 7850 DF, p-value: < 2.2e-16

For rank second airline:

- As the coefficient of FREQUENCY is 2.212*10^{-4}, fare will increase by 1.9 cents per mile when adding 1 flight per day.
- As the coefficient of FREQ_OTHERS is -3.249*10^{-5}, fare will decrease by 0.29 cents per mile when adding 1 flight per day.
- As the coefficient of SEATS is -1.023*10^{-6}, fare will decrease by 0.09 cents per mile when adding 10 seats per day.

(3) Three Main Carrier Market

There are 1,790 observations in the dataset, the results are:

Stage 1:

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	2.181e+00	1.537e-01	14.191	< 2e-16	***
logFUEL(t-1)_R1	1.568e-01	7.324e-02	2.141	0.032440	*
logLABOR_R1	5.011e-01	1.752e-01	2.860	0.004283	**
PASSENGERS_SUM	-2.781e-06	1.443e-06	-1.927	0.054162	.
GDP_CHANGE	7.706e-03	2.114e-03	3.645	0.000275	***
QUARTER1to4	2.670e-02	1.503e-02	1.776	0.075847	.
QUARTER2to4	2.888e-02	1.500e-02	1.925	0.054399	.
QUARTER3to4	1.382e-02	1.493e-02	0.926	0.354752	
TIMEZONE_absdiff_1to0	2.082e-01	1.850e-02	11.251	< 2e-16	***
TIMEZONE_absdiff_2to0	3.013e-01	4.836e-02	6.230	5.81e-10	***
TIMEZONE_absdiff_3to0	7.293e-01	1.182e-01	6.171	8.38e-10	***
DIST1	-2.819e-03	1.232e-04	-22.882	< 2e-16	***
DIST2	-1.144e-03	5.671e-05	-20.165	< 2e-16	***
DIST3	-3.870e-04	1.040e-04	-3.720	0.000205	***
FREQUENCY_R1	1.049e-04	3.838e-05	2.733	0.006336	**
SEATS_R1	-1.122e-06	3.390e-07	-3.308	0.000957	***
AAtoWN_R1	2.161e+00	3.243e-01	6.665	3.54e-11	***
DLtoWN_R1	2.043e+00	2.236e-01	9.138	< 2e-16	***
UAtoWN_R1	1.086e+00	2.229e-01	4.869	1.22e-06	***
UStoWN_R1	1.285e+00	5.551e-01	2.315	0.020711	*
logFUEL(t-1)_AA_R1	-7.977e-02	7.551e-02	-1.056	0.290887	
logFUEL(t-1)_DL_R1	-7.406e-02	7.404e-02	-1.000	0.317342	
logFUEL(t-1)_UA_R1	-5.573e-02	7.434e-02	-0.750	0.453563	
logFUEL(t-1)_US_R1	5.982e-03	8.652e-02	0.069	0.944886	
logLABOR_AA_R1	-4.686e-01	2.048e-01	-2.288	0.022263	*
logLABOR_DL_R1	-5.268e-01	1.851e-01	-2.847	0.004470	**
logLABOR_UA_R1	-1.678e-01	1.832e-01	-0.916	0.360024	
logLABOR_US_R1	-3.116e-01	2.221e-01	-1.403	0.160790	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2228 on 1762 degrees of freedom
Multiple R-squared: 0.8167, Adjusted R-squared: 0.8138
F-statistic: 290.7 on 27 and 1762 DF, p-value: < 2.2e-16

For rank first airline:

- As the coefficient of FREQUENCY is 1.049*10^{-4}, fare will increase by 0.95 cents per mile when adding 1 flight per day.
- As the coefficient of SEATS is -1.122*10^{-6}, fare will decrease by 0.1 cents per mile when adding 10

seats per day.

Stage 2:

```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)            2.086e-01  2.307e-01   0.904 0.365893
logAIRFAIR(t)_R1       7.581e-01  1.387e-02  54.671  < 2e-16 ***
logFUEL(t-1)_R2        1.243e-01  5.369e-02   2.316 0.020682 *
logLABOR_R2            8.507e-02  1.151e-01   0.739 0.459819
PASSENGERS_SUM        -4.582e-06  1.078e-06  -4.251 2.24e-05 ***
GDP_CHANGE             1.074e-03  1.321e-03   0.813 0.416068
QUARTER 1to4          -7.920e-03  9.247e-03  -0.856 0.391845
QUARTER 2to4           1.444e-02  9.233e-03   1.564 0.118055
QUARTER 3to4           7.787e-03  9.222e-03   0.844 0.398573
TIMEZONE_absdiff_1to0 -6.803e-02  1.205e-02  -5.647 1.90e-08 ***
TIMEZONE_absdiff_2to0 -1.215e-01  3.144e-02  -3.866 0.000114 ***
TIMEZONE_absdiff_3to0 -7.857e-01  7.166e-02 -10.964  < 2e-16 ***
DIST1                 -8.967e-04  7.717e-05 -11.620  < 2e-16 ***
DIST2                 -9.541e-05  3.712e-05  -2.570 0.010252 *
DIST3                  7.718e-04  6.617e-05  11.664  < 2e-16 ***
FREQUENCY_R2           4.852e-04  4.082e-05  11.886  < 2e-16 ***
FREQ_OTHERS_R2        -2.747e-05  6.543e-06  -4.198 2.82e-05 ***
SEATS_R2              -2.903e-06  3.398e-07  -8.543  < 2e-16 ***
AAtoWN_R2              1.276e+00  2.759e-01   4.625 4.02e-06 ***
DLtoWN_R2              2.468e-01  2.506e-01   0.985 0.324942
UAtoWN_R2              1.874e-01  2.560e-01   0.732 0.464270
UStoWN_R2              3.861e-01  3.600e-01   1.073 0.283589
logFUEL(t-1)_AA_R2    -7.882e-02  5.484e-02  -1.437 0.150790
logFUEL(t-1)_DL_R2    -4.526e-02  5.466e-02  -0.828 0.407780
logFUEL(t-1)_UA_R2     7.700e-04  5.468e-02   0.014 0.988766
logFUEL(t-1)_US_R2     5.741e-02  5.928e-02   0.968 0.332942
logLABOR_AA_R2        -2.947e-01  1.274e-01  -2.312 0.020870 *
logLABOR_DL_R2        -1.365e-02  1.197e-01  -0.114 0.909222
logLABOR_UA_R2         5.302e-02  1.215e-01   0.436 0.662645
logLABOR_US_R2        -1.947e-01  1.329e-01  -1.465 0.143022
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1372 on 1760 degrees of freedom
Multiple R-squared:  0.9413,    Adjusted R-squared:  0.9403
F-statistic: 972.6 on 29 and 1760 DF,  p-value: < 2.2e-16
```

For rank second airline:

- As the coefficient of FREQUENCY is $4.852*10^{-4}$, fare will increase by 4.46 cents per mile when adding 1 flight per day.
- As the coefficient of FREQ_OTHERS is $-2.747*10^{-5}$, fare will decrease by 0.25 cents per mile when adding 1 flight per day.
- As the coefficient of SEATS is $-2.903*10^{-6}$, fare will decrease by 0.26 cents per mile when adding 10 seats per day.

Stage 3:

```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)            1.155e+00  6.652e-01   1.736  0.08272 .
logAIRFAIR(t)_R1       4.263e-01  2.251e-02  18.934  < 2e-16 ***
logAIRFAIR(t)_R2       2.700e-01  2.053e-02  13.151  < 2e-16 ***
logFUEL(t-1)_R3        6.017e-02  1.169e-01   0.515  0.60679
logLABOR_R3            6.576e-03  3.581e-01   0.018  0.98535
PASSENGERS_SUM        -1.366e-06  1.106e-06  -1.236  0.21671
GDP_CHANGE             2.906e-03  1.386e-03   2.096  0.03620 *
QUARTER1to4            6.203e-03  9.905e-03   0.626  0.53124
QUARTER2to4            4.257e-04  9.857e-03   0.043  0.96556
QUARTER3to4            4.259e-03  9.840e-03   0.433  0.66524
TIMEZONE_absdiff_1to0  9.019e-02  1.343e-02   6.716 2.51e-11 ***
TIMEZONE_absdiff_2to0 -6.029e-02  3.430e-02  -1.758  0.07900 .
TIMEZONE_absdiff_3to0 -1.447e-01  7.805e-02  -1.854  0.06391 .
DIST1                 -1.369e-03  8.293e-05 -16.513  < 2e-16 ***
DIST2                 -2.013e-04  3.841e-05  -5.241 1.79e-07 ***
```

```
DIST3                      2.313e-04   7.376e-05    3.135   0.00174 **
FREQUENCY_R3               8.857e-04   6.660e-05   13.297   < 2e-16 ***
FREQ_OTHERS_R3             1.544e-05   6.931e-06    2.228   0.02599 *
SEATS_R3                  -5.193e-06   5.143e-07  -10.099   < 2e-16 ***
AAtoWN_R3                  2.702e-01   6.751e-01    0.400   0.68902
DLtoWN_R3                 -5.148e-01   6.700e-01   -0.768   0.44236
UAtoWN_R3                  4.781e-01   6.743e-01    0.709   0.47841
UStoWN_R3                 -2.744e-01   7.107e-01   -0.386   0.69947
logFUEL(t-1)_AA_R3        -1.701e-02   1.176e-01   -0.145   0.88500
logFUEL(t-1)_DL_R3         5.507e-02   1.176e-01    0.468   0.63970
logFUEL(t-1)_UA_R3         3.968e-02   1.174e-01    0.338   0.73544
logFUEL(t-1)_US_R3         2.884e-02   1.204e-01    0.240   0.81067
logLABOR_AA_R3            -1.057e-01   3.612e-01   -0.293   0.76983
logLABOR_DL_R3             4.334e-02   3.595e-01    0.121   0.90407
logLABOR_UA_R3            -2.249e-01   3.606e-01   -0.624   0.53294
logLABOR_US_R3             3.266e-02   3.619e-01    0.090   0.92811
---
Signif. codes:   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1463 on 1759 degrees of freedom
Multiple R-squared:   0.9326,    Adjusted R-squared:   0.9315

F-statistic: 811.8 on 30 and 1759 DF,   p-value: < 2.2e-16
```

For rank third airline:

- As the coefficient of FREQUENCY is $8.857*10^{-4}$, fare will increase by 8.3 cents per mile when adding 1 flight per day.
- As the coefficient of FREQ_OTHERS is $-1.544*10^{-5}$, fare will decrease by 0.14 cents per mile when adding 1 flight per day.
- As the coefficient of SEATS is $-5.193*10^{-6}$, fare will decrease by 0.47 cents per mile when adding 10 seats per day.

4.4 Discussing about the Airfare and Capacity Allocation

Using table 1, here lists the results above.

Table1. airfare and capacity allocation

		Airfare increase when adding 1flight per day (cents/passenger.mile)	Airfare decrease when other airline adding 1flight per day (cents/passenger.mile)	Airfare decrease when adding 10 seats per day(cents/passenger.mile)
The aggregate market		3.86	-0.002	0.13
Multi-stage market	One main airline	1.832	0.25	0.04
	Two main airline	5.77		0.19
		1.90	0.29	0.09
	Three main airline	0.95		0.10
		4.46	0.25	0.26
		8.30	0.14	0.47

As for the airline in market, table 1 implies:

- adding flight leads to its airfare increasing, but other airline adding flight leads to its airfare decreasing;
- adding flight has better effects than adding seats

- airline with high market share gets more advantage than lower one.

5 Case Study for One-Stop Domestic Market

6 Conclusion

References

- Adler, N. (2001). Competition in a Deregulated Air Transportation Market. *European Journal of Operational Research*. Vol. 129, No. 2, pp 337–345.
- Adler, N. (2005). Hub-Spoke Network Choice Under Competition with an Application to Western Europe. *Transportation Science*. Vol. 39, No. 1, pp 58–72.
- Adler, N., and J. Berechman (2001). Evaluating Optimal Multi-Hub Networks in a Deregulated Aviation Market with an Application to Western Europe. *Transportation Research Part A: Policy and Practice*. Vol. 35, pp 373–390.
- Adler, N., and K. Smilowitz (2007). Hub-and-Spoke Network Alliances and Mergers: Price-Location Competition in the Airline Industry. *Transportation Research Part B: Methodological*. Vol. 41, No. 4, pp 394–409.
- Adler, N., E. Pels, and C. Nash (2010). High-Speed Rail and Air Transport Competition: Game Engineering as Tool for Cost-Benefit Analysis. *Transportation Research Part B: Methodological*. Vol. 44, No. 7, pp 812–833.
- Aguirregabiria, V., C. Y. Ho (2012). A Dynamic Oligopoly Game of the US Airline Industry: Estimation and Policy Experiments. *Journal of Econometrics*. Vol. 168, No. 1, pp 156–173.
- Aguirregabiria, V., C. Y. Ho (2010). A Dynamic Game of Airline Network Competition: Hub-and-Spoke Networks and Entry Deterrence. *International Journal of Industrial Organization*. Vol. 28, No. 4, pp 377–382.
- Airlines for America (2015). Per-Minute Cost of Delays to U.S. Airlines. URL: http://airlines.org/data/per-minute-cost-of-delays-to-u-s-airlines/. Accessed on Aug 5th, 2015.
- Arabeyre, J. P., J. Fearnley, F. C. Steiger, and W. Teather (1969). The Airline Crew Scheduling Problem: A Survey. *Transportation Science*, Vol. 3, No. 2, 140–163.
- Atasoy, B., M. Salani, and M. Bierlaire (2014). An Integrated Airline Scheduling, Fleeting and Pricing Model for Monopolized Market. *Computer-Aided Civil and Infrastructure Engineering*, Vol. 29, No. 2, pp 76–90.
- Ball, M., C. Barnhart, M. Dresner, M. Hansen, K. Neels, A. Odoni, E. Peterson, L. Sherry, A. Trani, B. Zou, R. Britto, D. Fearing, P. Swaroop, N. Umang, V. Vaze, and A. Voltes (2010). Total Delay Impact Study: A Comprehensive Assessment of the Costs and Impacts of Flight Delay in the United States. Final Report, *The National Center of Excellence for Aviation Operations Research*.

- Barnhart, C., D. Fearing, A. Odoni, and V. Vaze (2012). Demand and Capacity Management in Air Transportation. *EURO Journal on Transportation and Logistics*, Vol. 1, No. 1/2, pp 135–155.
- Barnhart, C., D. Fearing, and V. Vaze (2014). Modeling Passenger Travel and Delays in the National Air Transportation System. *Operations Research*. Vol. 62, No. 3, pp 580–601.
- Barnhart, C., E. L. Johnson, G. L. Nemhauser, M. W. P. Salvesbergh, and P. H. Vance (1998). Branch-and-Price: Column Generation for Solving Huge Integer Programs. *Operations Research*. Vol. 46, No. 3, pp 316–329.
- Barnhart, C., T. Kniker, and M. Lohatepanont (2002). Itinerary-Based Airline Fleet Assignment. *Transportation Science*. Vol. 36, No. 2, pp 199–217.
- Belobaba, P. P. (1989). OR Practice–Application of a Probabilistic Decision Model to Airline Seat Inventory Control. *Operations Research*. Vol. 37, No. 2, pp. 183–197.
- Belobaba, P. (2015a). Airline Revenue Management. In *The Global Airline Industry, 2nd Edition*, ed., P. Belobaba, A. Odoni, and C. Barnhart, pp. 99–126. John Wiley & Sons, Inc., New York. ISBN: 978–1–118–88117–0.
- Belobaba, P. (2015b). The Airline Planning Process. In *The Global Airline Industry, 2nd Edition*, ed., P. Belobaba, A. Odoni, and C. Barnhart, pp. 159–188. John Wiley & Sons, Inc., New York. ISBN: 978–1–118–88117–0.
- Barnhart, C., and V. Vaze (2015). Airline Schedule Optimization. In *The Global Airline Industry, 2nd Edition*, ed., P. Belobaba, A. Odoni, and C. Barnhart, pp. 189–222. John Wiley & Sons, Inc., New York. ISBN: 978–1–118–88117–0.
- Bhadra, D., J. Gentry, B. Hogan, and M. Wells (2005). Future Air Traffic Timetable Estimator. *Journal of Aircraft*. Vol. 42, No. 2, pp 320–328.
- Blaiberg, J. (2013). *An Integrated Game-Theoretic Airline Schedule Planning Model with Embedded Spill and Recapture*. MS Thesis, Swiss Federal Institute of Technology, Lausanne, Switzerland.
- Bot, R. I., S. M. Grad, and G. Wanka (2006). Almost Convex Functions: Conjugacy and Duality. In *Generalized Convexity and Related Topics*, Ed. I. V. Konnov, D. T. Luc, and A. M. Rubinov. Lecture Notes in Economics and Mathematical Systems. Springer-Verlag, Berlin, Germany. ISBN: 978–3–540–37006–2.
- Boyd, S., and L. Vandenberghe (2004). Convex Optimization. Cambridge University Press, Cambridge, United Kingdom. ISBN: 978–0–521–83378–3.
- Brander, J. A., and A. Zhang (1993). Dynamic Oligopoly Behavior in the Airline Industry. *International Journal of Industrial Organization*. Vol. 11, No. 3, pp 407–435.
- Brueckner, J. K. (2002). Airport Congestion When Carriers Have Market Power. *American Economic Review*. Vol. 92, No. 5, pp 1357–1375.
- Brueckner, J. K. (2010). Schedule Competition Revisited. *Journal of Transport Economic Policy*. Vol. 44, No. 3, pp 261–285.
- Brueckner, J.K., and R. Flores-Fillol (2007). Airline Schedule Competition. *Review of Industrial Organization*. Vol. 30, No. 3, pp 161–177.

- BTS (2015a). Air Carrier Statistics Database (T100). Office of the Assistant Secretary for Research and technology, Bureau of Transportation Statistics, United States Department of Transportation, Washington D.C. URL: http://www.transtats.bts.gov/DatabaseInfo.asp?DB_ID=111. Accessed on Aug 15th, 2015.
- BTS (2015b). Airline Origin Destination Survey (DB1B). Office of the Assistant Secretary for Research and technology, Bureau of Transportation Statistics, United States Department of Transportation, Washington D.C. URL: http://www.transtats.bts.gov/DatabaseInfo.asp?DB_ID=125&DB_Name=Airline%20Origin%20and%20Destination%20Survey%20(DB1B). Accessed on Aug 15th 2015.
- Button, K., and J. Drexler (2005). Recovering Costs by Increasing Market Share: An Empirical Critique of the S-Curve. *Journal of Transport Economics and Policy*. Vol. 39, No. 3, pp 391–404.
- Cadarso, L., V. Vaze, C. Barnhart, and A. Marin (2015). Integrated Airline Scheduling: Considering Competition Effects and the Entry of the High Speed Rail. Accepted. *Transportation Science*.
- Cambini, A., and L. Martein (2009). Generalized Convexity and Optimization: Theory and Applications. Lecture Notes in Economics and Mathematical Systems. Springer-Verlag, Berlin, Germany. ISBN: 978–3–540–70875–9.
- Caragiannis, I., A. Fanelli, N. Gravin, and A. Skopalik (2012). Computing Approximate Pure Nash Equilibria in Congestion Games. *ACM SIGecom Exchanges*. Vol. 11, No. 1, pp 26–29.
- Clough, M. C., T. L. Jacobs, and E. S. Gel (2014). A Choice-Based Mixed Integer Programming Formulation for Network Revenue Management. *Journal of Revenue and Pricing Management*. Vol. 13, pp, 366–387.
- Cohn, A., and C. Barnhart (2003). Improving Crew Scheduling by Incorporating Key Maintenance Routing Decisions. *Operations Research*, Vol. 51, No. 3, pp 387–396.
- Cook, A., G. Tanner, and S. Anderson (2004). Evaluating the True Cost to Airlines of One Minute of Airborne or Ground Delay: Final Report. *Performance Review Unit, EUROCONTROL*. Brussels, Belgium.
- Cordeau, J., G. Stojkovic, F. Soumis, and J. Desrosiers (2001). Benders Decomposition for Simultaneous Aircraft Routing and Crew Scheduling. *Transportation Science*, Vol. 35, No. 4, pp 375–388.
- Desaulniers, G., J. Desrosiers, Y. Dumas, M. Solomon, and F. Soumis (1997). Daily Aircraft Routing and Scheduling. *Management Science*. Vol. 43, No. 6, pp 841–855.
- Dobson, G., and P. J. Lederer (1993). Airline Scheduling and Routing in a Hub-and-Spoke System. *Transportation Science*. Vol. 27, No. 3, pp 281–297.
- Evans, A., V. Vaze, and C. Barnhart (2014). Airline-Driven Performance-Based Air Traffic Management: Game Theoretic Models and Multi-criteria Evaluation. *Transportation Science*. Articles in Advance, pp 1–24.
- FAA (2015). Terminal Area Forecast. Federal Aviation Administration, Washington, D.C. URL: https://aspm.faa.gov/main/taf.asp. Accessed on Aug 15th, 2015.
- Garrow, L. A. (2012). Discrete Choice Modelling and Air Travel Demand Theory and Applications. Ashgate Publishing, Surrey, United Kingdom. ISBN: 978–1–4094–8633–6.

- Hane, C. A., C. Barnhart, E. L. Johnson, R. E. Marsten, G. L. Nemhauser, and G. Sigismondi (1995). *Mathematical Programming*. Vol. 70, No. 1–3, pp 211–232.
- Hansen, M., and Y. Liu (2015). Airline Competition and Market Frequency: A Comparison of the S-Curve and Schedule Delay Models. *Transportation Research Part B: Methodological*. Vol. 78, pp 301–317.
- Hansen, M. (1990). Airline Competition in a Hub-Dominated Environment: An Application of Non-Cooperative Game Theory. *Transportation Research Part B: Methodological*. Vol. 24, No. 1, pp 27–43.
- Hansen, N., A. Auger, R. Ros, S. Finck, and P. Posik (2010). Comparing Results of 31 Algorithms from the Black-Box Optimization Benchmarking BBOB-2009. Proceedings of the 12th Annual Conference Companion on Genetic and Evolutionary Computation (GECCO '10), pp 1689–1696.
- Harder, R., and V. Vaze (2015). *Two-Stage Game Theoretic Modeling of Airline Frequency and Fare Competition*. Working Paper. ADD LINK.
- Helble, J. J. (2015). Approaching Gender Parity. *Dartmouth Engineering Magazine*, Spring 2015. Thayer School of Engineering, Hanover, NH. URL: https://engineering.dartmouth.edu/assets/pdf/dartmouth-engineer-spring-2015.pdf. Accessed on Aug 15th, 2015.
- Hendricks, K., M. Piccione, and G. Tan (1999). Equilibria in Networks. *Econometrica*. Vol. 67, No. 6, pp 1407–1434.
- Hong, S., and P. T. Harker (1992). Air Traffic Network Equilibrium: Toward Frequency, Price and Slot Priority Analysis. *Transportation Research Part B: Methodological*. Vol. 26B, No. 4, pp 307–323.
- Jacobs, T. L., B. C. Smith, and E. L. Johnson (2008). Incorporating Network Flow Effects into the Airline Fleet Assignment Process. *Transportation Science*, Vol. 42, No. 4, pp 514–529.
- Jacquillat, A., and V. Vaze (2015). Inter-Airline Equity and Airline Collaboration in Airport Scheduling Interventions. *Operations Research*. Under Review.
- Kahn A. E. (1993). Change, Challenge, and Competition: A Review of the Airline Commission Report. *Regulation*, Vol. 16, pp 55–64.
- Klabjan, D., E. L. Johnson, and G. L. Nemhauser (2001). Solving Large Airline Crew Scheduling Problems: Random Pairing Generation and Strong Branching. *Computational Optimization and Applications*, Vol. 20, No. 1, pp 73–91.
- Konnov, I. V., D. T. Luc, and A. M. Rubinov (2006). Generalized Convexity and Related Topics. Lecture Notes in Economics and Mathematical Systems. Springer-Verlag, Berlin, Germany. ISBN: 978–3–540–37006–2.
- Li, X., and V. Vaze (2015). *Hybrid Models of Airline Competition: Capacity Allocation Games with Embedded Airfare Regression*. Working Paper. ADD LINK.
- Liu, Y., and M. Hansen (2015). Airline Competition and Market Frequency: A Comparison of the S-Curve and Schedule Delay Models. *Transportation Research Part B: Methodological*. Vol. 78, pp 301–317.

- Lohatepanont, M., and C. Barnhart (2004). Airline Schedule Planning: Integrated Models and Algorithms for Schedule Design and Fleet Assignment. *Transportation Science*, Vol. 38, No. 1, pp 19–32.
- McGill, J. I., and G. J. Van Ryzin (1999). Revenue Management: Research Overview and Prospects. *Transportation Science*. Vol. 33, No. 2, pp. 233–256.
- Njuguna, E. N. (2014). *A Longitudinal Analysis of Passenger Travel Disruptions in the National Air Transportation System.* M.S. Thesis, Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Cambridge, MA.
- Norman, V. D., and S. P. Strandenes (1994). Deregulation of Scandinavian Airlines: A Case Study of the Oslo-Stockholm Route. Krugman P, Smith A, eds. *Empirical Studies of Strategic Trade Policy* (University of Chicago Press, Chicago), pp 85–100.
- O'Connor, W. E. (2001). An Introduction to Airline Economics. Greenwood Publishing Group, Westport, CT. ISBN: 0–275–96911–8.
- Odoni, A., and T. Morisset (2011). Capacity, Delay and Schedule Reliability. *Transportation Research Record: Journal of the Transportation Research Board*. Vol. 2214, pp 85–93.
- Pels, E., P. Nijkamp, and P. Rietveld (2000). Airport and Airline Competition for Passengers Departing from a Large Metropolitan Area. *Journal of Urban Economics*. Vol. 48, No. 1, pp 29–45.
- Pita, J. P., C. Barnhart, and A. P. Antunes (2013). Integrated Flight Scheduling and Fleet Assignment Under Airport Congestion. *Transportation Science*. Vol. 47, No. 4, pp 477–492.
- Pita, J. P., A. P. Antunes, C. Barnhart, and A. G. Menezes (2013). Setting Public Service Obligations in Low-Demand Air Transportation Networks: Applications to the Azores. *Transportation Research Part A: Policy and Practice*. Vol. 54, pp 35–48.
- Pita, J. P., N. Adler, and A. P. Antunes (2014). Socially-Oriented Flight Scheduling and Fleet Assignment Model with an Application to Norway. *Transportation Research Part B: Methodological*. Vol. 61, pp 17–32.
- Rexing, B., C. Barnhart, T. Kniker, A. Jarrah, and N. Krishnamurthy (2000). Airline Fleet Assignment with Time Windows. *Transportation Science*, Vol. 34, No. 1, pp 1–20.
- Rosen, J. B. (1965). Existence and Uniqueness of Equilibrium Points for Concave N-Person Games. *Econometrica*. Vol. 33, No. 3, pp 520–534.
- Saddoune, M., G. Desaulniers, I. Elhallaoui, and F. Soumis (2012). Integrated Airline Crew Pairing and Crew Assignment by Dynamic Constraint Aggregation. *Transportation Science*, Vol. 46, No. 1, pp 39–55.
- Schipper, Y., P. Rietveld, and P. Nijkamp (2003). Airline Deregulation and External Costs: A Welfare Analysis. *Transportation Research Part B: Methodological*. Vol. 37, No. 8, pp 699–718.
- Schumer, C. E., and C. B. Maloney (2008). Your Flight Has Been Delayed Again: Flight Delays Cost Passengers, Airlines and The U.S. Economy Billions. *U.S. Senate Joint Economic Committee*, Washington, D.C.

- Sen, S., C. Barnhart, J. R. Birge, E. A. Boyd, M. C. Fu, D. S. Hochbaum, D. P. Morton, G. L. Nemhauser, B. L. Nelson, W. B. Powell, C. A. Shoemaker, D. D. Yao, and S. A. Zenios (2014). O.R. as a Catalyst for Engineering Grand Challenges. Report to the *National Science Foundation*, Arlington, VA.
  URL: http://connect.informs.org/communities/community-home/librarydocuments/viewdocument/?DocumentKey=d7e454e3-1872-4826-900b-7871063a5980.
- Sherali, H. D., K. H. Bae, and M. Haouari (2010). Integrated Airline Schedule Design and Fleet Assignment: Polyhedral Analysis and Benders' Decomposition Approach. *INFORMS Journal of Computing*, Vol. 22, No. 4, pp 500–513.
- Sherali, H. D., K. H. Bae, and M. Haouari (2013). An Integrated Approach for Airline Flight Selection and Timing, Fleet Assignment, and Aircraft Routing. *Transportation Science*, Vol. 47, No. 4, pp 455–476.
- Swan, W., and N. Adler (2006). Aircraft Trip Cost Parameters: A Function of Stage Length and Seat Capacity. *Transportation Research Part E: Logistics and Transportation Review*. Vol. 42, No. 2, pp 105–115.
- Talluri, K.T., and G. J. van Ryzin (2004). The Theory and Practice of Revenue Management, International Series in Operations Research and Management Science. Kluwer Academic Press, Boston. ISBN: 978–0–387–27391–4.
- Vaze, V., and C. Barnhart (2012a). Modeling Airline Frequency Competition for Airport Congestion Mitigation. *Transportation Science*. Vol. 46, No. 4, pp 512–535.
- Vaze, V., and C. Barnhart (2012b). The Role of Airline Frequency Competition in Airport Congestion Pricing. *Transportation Research Record*, Vol. 2226, pp 69–77.
- Vaze, V., and C. Barnhart (2015). Price of Airline Frequency Competition. *Game-Theoretic Analysis of Congestion, Safety and Security* (Eds. K. Hausken and J. Zhuang). Springer Series in Reliability Engineering. ISBN: 978–3–319–13008–8.
- Wang, D. D., D. Klabjan, and S. Shebalov (2014). Attractiveness-Based Airline Network Models with Embedded Spill and Recapture. *Journal of Airline and Airport Management*, Vol. 4, No. 1, pp 1–25.
- Wei, W., and M. Hansen (2005). Impact of Aircraft Size and Seat Availability on Airlines' Demand and Market Share in Duopoly Markets. *Transportation Research Part E: Logistics and Transportation Review*. Vol. 41, No. 4, pp 315–327.
- Wei, W., and M. Hansen (2006). An Aggregate Demand Model for Air Passenger Traffic in the Hub-and-Spoke Network. *Transportation Research Part A: Policy and Practice*. Vol. 40, No. 10, pp 841–851.
- Wei, W., and M. Hansen (2007). Airlines' Competition in Aircraft Size and Service Frequency in Duopoly Markets. *Transportation Research Part E: Logistics and Transportation Review*. Vol. 43, No. 4, pp 409–424.
- Wittman, M. D. (2014). *The Effects of Capacity Discipline on Smaller U.S. Airports: Trends in Service, Connectivity, and Fares*. M.S. Thesis, Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Cambridge, MA.
- Harder, Wei and Vaze (2015): Add links to the demos.